About orbits of a neuron model with a periodic internal decay rate

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In [1] a difference equation $x_{n+1} = \beta x_n - g(x_n)$, $n = 0, 1, 2, ..., $ was analyzed as a single neuron model, where $\beta > 0$ is an internal decay rate and a signal function $g$ is the following piecewise constant function with McCulloch-Pitts nonlinearity:

$$g(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases}$$

(1)

Similar models of one, two or three neurons with constant internal decay rate $\beta$ were studied in many articles. In [2] authors analyze one neuron model with signal function with three thresholds (not only 0) and obtain interesting dynamics of solutions. Here we will investigate the periodic character of solutions relative to the periodic internal decay rate; in particular, we will study the following non-autonomous piecewise linear difference equation:

$$x_{n+1} = \beta_n x_n - g(x_n), \quad n = 0, 1, 2, ...,$$

where

$$\beta_n = \begin{cases} \beta_0, & \text{if } n \text{ is even} \\ \beta_1, & \text{if } n \text{ is odd} \end{cases}$$

and $g$ is in form (1). Our goal is to study the periodic orbits and their basin of attraction. These results have been published in [3].


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