Relationship between the root -1 of a characteristic equation and period-two solutions[1]

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In the book [1] many open problems and conjectures about third-order rational difference equations have been formulated. Some of them are related to the relationship between the root -1 of a characteristic equation of linearized equation of some difference equation and period-two solutions of the same difference equation. In article [2] we investigated three second-order rational difference equations with period-two solutions and found that the characteristic equations of these difference equations have a root -1. We try to clarify the relationship between the root -1 and period-two solutions of difference equations.
For example, a characteristic equation of a second-order linear difference equation

$$x_{n+1} = Ax_n + (A+1)x_{n+1}$$

has a root -1. If $A \in]-2, 0[$, then every pair of initial conditions in the form x_{-1}, $x_0 = -x_{-1}$ makes a periodic solution with period two. But a solution with an arbitrary chosen initial conditions converges to a period-two solution in the form

$$\left\{ \frac{x_{-1}(A+1) - x_0}{A+2}, \frac{x_0 - x_{-1}(A+1)}{A+2} \right\}.$$

In our presentation we consider linear difference equations and some rational difference equations.

[1] Camouzis, E., Ladas, G., Dynamics of Third-Order Rational Difference Equations with Open Problems and Conjectures. *Chapman and Hall/CRC*, USA, 2008.

[2] Avotina, M., On Three Second-Order Rational Difference Equations with Period-Two Solutions, *Int. J. Difference Equ.* 9 (2014), 23–35.

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