Relationship between the root \(-1\) of a characteristic
equation and period-two solutions

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In the book [1] many open problems and conjectures about third-order rational
difference equations have been formulated. Some of them are related to the relation-
ship between the root \(-1\) of a characteristic equation of linearized equation of
some difference equation and period-two solutions of the same difference equation.
In article [2] we investigated three second-order rational difference equations with
period-two solutions and found that the characteristic equations of these difference
equations have a root \(-1\). We try to clarify the relationship between the root \(-1\)
and period-two solutions of difference equations.
For example, a characteristic equation of a second-order linear difference equation
\[ x_{n+1} = Ax_n + (A + 1)x_{n+1} \]
has a root \(-1\). If \(A \in ]-2,0[\), then every pair of initial conditions in the form
\(x_{-1}, x_0 = -x_{-1}\) makes a periodic solution with period two. But a solution with an
arbitrary chosen initial conditions converges to a period-two solution in the form
\[ \left\{ \frac{x_{-1}(A + 1) - x_0}{A + 2}, \frac{x_0 - x_{-1}(A + 1)}{A + 2} \right\}. \]

In our presentation we consider linear difference equations and some rational diffe-
rence equations.

[1] Camouzis, E., Ladas, G., Dynamics of Third-Order Rational Difference Equa-

Two Solutions, Int. J. Difference Equ. 9 (2014), 23–35.

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