Uniform asymptotic stability and exponential stability for two-dimensional half-linear difference systems

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We consider a half-linear difference system:

$$\Delta x_k = a_k x_k + b_k \phi_{p^*}(y_k), \quad \Delta y_k = c_k \phi_p(x_k) + d_k y_k,$$

where all coefficients are real-valued sequences; p and p^* are positive numbers satisfying $1/p + 1/p^* = 1$; and $\phi_q(z) = |z|^{q-2}z$ for $q = p$ or $q = p^*$. A differential system corresponding to the half-linear difference system appeared in [2]. In the special case that $p = 2$, this system becomes the two-dimensional linear difference system $\Delta \mathbf{x}_k = A_k \mathbf{x}_k$ where A_k is a 2×2 matrix. It is known that uniform asymptotic stability implies exponential stability for the linear difference systems (see [1]). However, in general, uniform asymptotic stability does not imply exponential stability in the case of nonlinear systems. The aim of this talk is to clarify that uniform asymptotic stability, exponential stability, and global exponential stability are equivalent for the half-linear difference system.

[1] Michel, A.N., Hou, L., Liu, D., *Stability of dynamical systems: Continuous, discontinuous, and discrete systems*, Systems & Control: Foundations & Applications. Birkhäuser Boston, Inc., Boston, MA, 2008.

[2] Onitsuka, M., Soeda, T., Uniform asymptotic stability implies exponential stability for nonautonomous half-linear differential systems, *Adv. Difference Equ.* 2015, 2015:158, 24pp.