Continuous-time difference equations and dynamical systems

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We present the recently published book "Difference Equations with Continuous Argument" by E.Yu.Romanenko (National Academy of Sciences of Ukraine, Proc. Inst. Mathematics, vol. 100, 2014, in Russian), which summed up the research of Kyiv school of thought on dynamical systems in the field of continuous-time difference equations. The book is the first complete exposition of the qualitative theory of nonlinear difference equations

$$x(t+1) = f(x(t)), \quad t \in R^+,$$

with f being a continuous map of an interval ($\subseteq R$) into itself. These equations possess a surprising broad spectrum of solutions, ranging from stationary to quasi-random solutions behaving like stochastic processes when time is large. All complexities in the dynamics of nonlinear one-dimensional maps transform into an extremely complex structure of solutions, that becomes increasingly more intricate over time. This implies the non-standard behavior of solutions:

- asymptotic discontinuity – abrupt shifts over time,
- fractal geometry of graphs – strong difference from simple curves, right up to likeness to space-filling curves,
- unpredictability – intrinsic uncertainty on large time scales,
- self-stochasticity – asymptotically exact describing with stochastic processes.

Analysis is based on going to the infinite-dimensional dynamical system induced by the equation on the space of its initial states. This method is of frequent use in evolutionary problems theory, but here we come up against an obstacle: The associated dynamical system usually has no attractor in its phase space. Overcoming this obstacle and identifying features of solutions, which it causes, take a central place in the book.

These findings are also of applied significance: There are ample classes of boundary value problems for partial differential equations, which can be reduced to continuous-time difference equations, and the dynamical properties of the latter allow us to offer fundamentally novel scenarios for spatio-temporal chaos (cascade emergence of coherent structures, chaotic mixing, intermittency). We believe that continuous-time difference equations will be an effective tool for the modeling of complex nonlinear processes (including turbulence) in parameter-distributed systems.