Three types of stability conditions
for linear delay difference equations

Jan Čermák

Institute of Mathematics, Brno University of Technology
Technická 2, 616 69 Brno
Czech Republic
cermak.j@fme.vutbr.cz

This contribution discusses the problem of necessary and sufficient conditions for the asymptotic stability of the zero solution of the three-term linear difference equation

\[ y(n) = \alpha y(n - m) + \beta y(n - k), \quad n = 0, 1, 2, \ldots, \] (1)

where \( \alpha, \beta \) are real scalars and \( k > m \) are positive integers. Stability analysis of (1) and its particular cases is connected with the names of several outstanding mathematicians (such as F. Dannan, S. Elaydi, M. Kipnis, S. Kuruklis and G. Ladas) who significantly contributed to this topic and formulated two types of stability conditions. Our aim is to present the third type of necessary and sufficient conditions guaranteeing the asymptotic stability of the zero solution of (1). We show that, contrary to the existing types, this third type of stability conditions is fully explicit with respect to delays \( k \) and \( m \). Various comparisons, examples and illustrations will be also given to support theoretical results.