Geometry and Global Stability of Higher Dimensional Monotone Maps

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We discuss a new notion of monotonicity for maps on $\mathbb{R}^k$, called normal monotonicity, that has recently been introduced in [1] and builds on previous work of the authors in [2]. The new definition of monotonicity extends the classical notion of competitive planar maps of Smith [3] and geometrically captures the dynamics of a competitive system in higher dimensions. Namely, a map $F : \mathbb{R}^k \to \mathbb{R}^k$ is monotone at $p$ if for any hypersurface $\Gamma$ containing $p$ with $\eta(\Gamma(p)) > 0$, we have $\eta(F(\Gamma)) > 0$. Here $\eta$ denotes the normal vector at a hypersurface. Our main result is to show global stability for monotone maps that have a unique coexistence fixed point.


\(^1\)Joint work with Saber Elaydi and Rafael Luís.