Asymptotic Growth and Fluctuation in Sublinear Volterra Integro-differential Equations.

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We study the asymptotic behaviour of solutions of the integro-differential equation

$$x'(t) = \int_{[0,t]} \mu(ds) f(x(t-s)),\ t > 0;\ x(0) = \xi > 0,$$

μ is a positive measure and f is a positive function obeying $\lim_{x\to\infty} f(x)/x = 0$. In this framework this equation has a unique continuous solution on $[0, \infty)$ obeying $\lim_{t\to\infty} x(t) = \infty$ [1]. Using comparison methods we characterise the rate of growth of solutions in terms of related ordinary differential equations. With $\mu(\mathbb{R}^+) < \infty$ we obtain general results for asymptotically monotone nonlinearities. We discuss applications of these results to regularly varying nonlinearities and show how the theory of regular variation can be used to compensate for a lack of monotonicity in the case of bounded delay [2, 3]. Furthermore, under mild conditions on the forcing function, it is possible to extend the aforementioned theory to incorporate state-independent additive noise. We will also examine extensions of these results to Itô stochastic functional differential equations.

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