

Scrambled pairs

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A subset S of a topological dynamical system (X, f) containing at least two points is called a *scrambled set* if for any $x, y \in S$ with $x \neq y$ one has

$$\liminf_{n \rightarrow \infty} d(f^n(x), f^n(y)) = 0 \quad \text{and} \quad \limsup_{n \rightarrow \infty} d(f^n(x), f^n(y)) > 0,$$

d being the metric on X . Every such pair (x, y) is called a *scrambled pair*. The notions of scrambled pairs and scrambled sets and their role in topological dynamics will be discussed. The cardinality of scrambled sets will be studied and the question when the existence of one scrambled pair implies the existence of a large scrambled set will be addressed. The attention will be paid to the topological size of scrambled sets and to the dependence of it on the phase space or on the dynamics of the system. Some kinds of chaos based on scrambled pairs, in particular generic chaos, will be discussed. Some generalizations and recent directions of related research will be mentioned.