

Equilibrium Character of a Piecewise Linear System of Difference Equations

Wirot Tikjha^{1,4}, Evelina G. Lapierre² and Yongwimon
Lenbury^{3,4}

¹Faculty of Science and Technology, Pibulsongkram Rajabhat
University, Phitsanulok, Thailand.

²Department of Mathematics, Johnson and Wales University, 8
Abbott Park Place, 02903, Providence, RI, USA.

³Department of Mathematics, Mahidol University, Rama 6 Road,
Bangkok, Thailand.

⁴Centre of Excellence in Mathematics, PERDO, Commission on
Higher Education, Ministry of Education, Bangkok, Thailand.

wirottik@psru.ac.th, rlapierre2@verizon.net,
scylb@mahidol.ac.th

Grove and his team have been working on the following family of systems of piecewise linear difference equations: $x_{n+1} = |x_n| + ay_n + b$ and $y_{n+1} = x_n + cy_n + d$, where $a, b, c, d \in \{-1, 0, 1\}$ and $(x_0, y_0) \in \mathbf{R}^2$. They found that the behavior of solutions of the piecewise linear system of difference equations [1], $x_{n+1} = |x_n| - y_n - 1$ and $y_{n+1} = x_n + y_n$, is eventually periodic with period 3 for every initial condition $(x_0, y_0) \in \mathbf{R}^2$ and they have published various results. See [1, 2, 3]. We would like to share our results of : $x_{n+1} = |x_n| - y_n$ and $y_{n+1} = x_n - |y_n| - d$, d being any positive real number and the initial condition (x_0, y_0) is an element of \mathbf{R}^2 , which is the generalization of system $x_{n+1} = |x_n| - y_n$ and $y_{n+1} = x_n - |y_n| - 1$ [2]. After observation via computer program and some direct computations, we found that the equilibrium point of the system is $(d, 0)$. We show that the solution of the difference equation is eventually equilibrium point regardless of initial point in \mathbf{R}^2 . Moreover, the solutions of the system will reach the equilibrium point in 6 iterations. We prove it by finding 5 conditions and show that each of the 5 conditions is true for all real numbers x_0 and y_0 .

[1] E.A. Grove, E. Lapierre, W. Tikjha, On the Global Behavior of $x_{n+1} = |x_n| - y_n - 1$ and $y_{n+1} = x_n + |y_n|$, *Cubo Mathematical Journal*, 14(2012)125 - 166.

[2] Lapierre E.G., On the global behavior of some systems of difference equation, *Doctoral dissertation of University of Rhode Island* (2013), 87 - 92.

[3] Tikjha, W., Lapierre, E.G., Lenbury, Y., On the Global Character of the System of Piecewise Linear Difference Equations $x_{n+1} = |x_n| - y_n - 1$ and $y_{n+1} = x_n - |y_n|$, *Advances in Difference Equations*, Article ID 573281 (2010).

